Earthquake Engineering Course 2011

Structural Reliability

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CONTENTS

Structural Engineering is the art of molding materials we do not really understand, into shapes we cannot really analyze, so as to withstand forces we cannot really assess, in such a way that the public does not really suspect.

Ross Corotis

1. Probabilistic Concept

(1). Quantification of the safety

A decision made by an expert on the safety of a structure influences the safety of other individuals such as the life or the property. Therefore the openness and transparency of decision process are generally required. In order to satisfy such requirements, the quantification of safety is necessary. In particular a quantitative measure for the safety is introduced to find a solution as the balance between the economy, the environmental impacts and the efficiency. ISO2394 [1] was produced to provide such a measure for the structural safety in order to eliminate the tax barrier for the world-wide trade.

For infra-structures including such as bridges, tunnels, dams and buildings, on which any future phenomena will influence, only probabilistic estimation of future events is possible as nobody knows what happens in future in a definite way. In other words a probabilistic measure should be used for the quantitative safety for structures.

The probability is a concept for quantitative evaluation of uncertain physical property. It is convenient for the evaluation of environment or safety over time or space. When the probabilistic evaluation is supported by statistical data, the model is considered consistent. However even if there is not sufficient statistical information, experts can provide reasonable models based on their subjective judgments. Sufficient number of data are generally not possible for rare events such as earthquakes. Therefore experts are always responsible for probabilistic models for safety evaluations.

(2). Fundamentals for the probability

When X is a random variable, the cumulative distribution function can be defined accordingly.

$$F_x(x) = \Pr[X \le x] \qquad (1)$$

The probability density function is the derivative of the cumulative distribution function with respect to x.

$$f_x(x) = \frac{dF_x(x)}{dx} \qquad (2)$$

Representative values which characterize these functions are introduced. They are the mean (the first moment), mode and median and the variance (the second moment), the standard deviation and the coefficient of variation.

mean: $$\mu_x = \int_{-\infty}^{\infty} x f_X(x)dx \qquad (3a)$$

mode: $\hat{x}$ $\quad \dfrac{df_X(x)}{dx}\Big|_{x=\hat{x}} = 0$ (3b)

median $\tilde{x}$ $\quad F_X(\tilde{x}) = 0.5$ (3c)

and

variance $\quad {\sigma_X}^2 = \int_{-\infty}^{\infty} (x-\mu_X)^2 f_X(x)dx$ (4)

standard deviation σ_X

coefficient of variation (c.o.v.) $\quad V_X = \dfrac{\sigma_X}{\mu_X}$ (5)

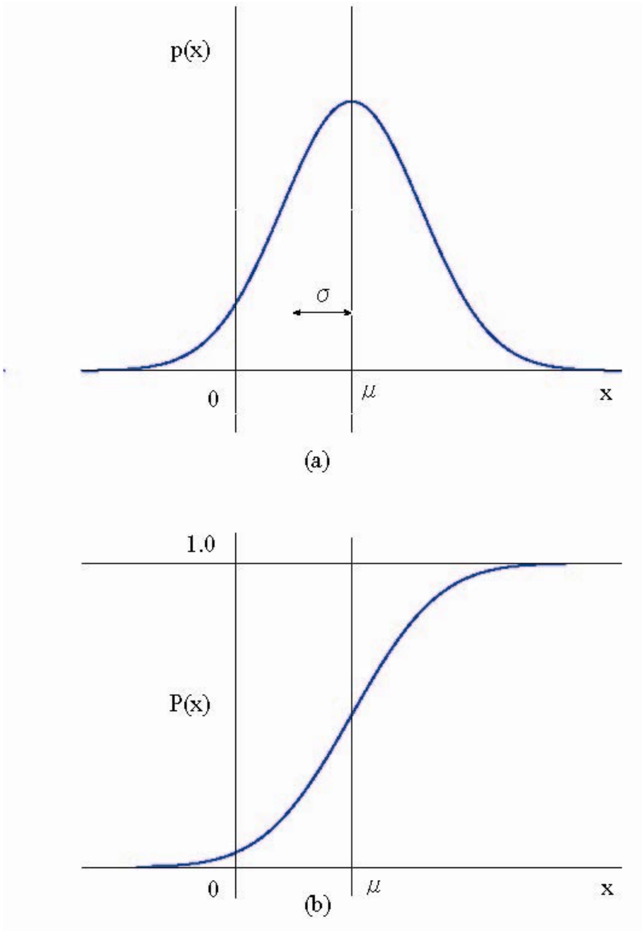

Figure 1 (a) Probability density function and (b) cumulative distribution function

(3). Useful probability distributions

The Gaussian distribution is a common example for the probability model and also known as the normal distribution.

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(\mu - x)^2}{2\sigma^2}} \tag{6}$$

In Gaussian distribution the mean μ and the standard deviation σ are all parameters necessary to describe the distribution. A special case with $\mu = 0$ and $\sigma = 1$ is known as the standard normal distribution and the density function is written as,

$$\phi(s) = \frac{1}{\sqrt{2\pi}} e^{-\frac{s^2}{2}} \tag{7}$$

Another common distribution used in the structural engineering is the log-normal distribution, where the logarithm of x is normally distributed.

$$p(x) = \frac{1}{\sqrt{2\pi}\zeta x} e^{-\frac{(\lambda - \ln x)^2}{2\zeta^2}} \tag{8}$$

where λ is the mean of $\ln x$ and ς is the standard deviation of $\ln x$. When ς is sufficiently less than 1, the following approximation is also useful.

$$\lambda = \ln \mu - \frac{1}{2}\varsigma^2 \quad \text{and} \quad \varsigma^2 = \ln(1 + \frac{\sigma^2}{\mu^2}) \tag{9}$$

(4). Reliability index and probability of failure

The safety means that the structure does not fail in a period of interest. By simple definition of being safety is that the resistance of structure exceeds the load effects, which are the responses of structure as consequences of loads acting on the structure.

The definition of probability of failure is written as the definition.

$$P_f = \Pr[z < 0] = \Pr[R \le Q] \tag{10}$$

where R is the resistance and Q is the load effects and $z = R - Q$. Then the reliability index can be defined as using the mean μ and the standard deviation σ as,

$$\beta = \frac{\mu_z}{\sigma_z} = \frac{\mu_r - \mu_q}{\sqrt{\sigma_r^2 + \sigma_q^2}} \tag{11}$$

When R and Q are variables of Gaussian distribution, the integral of equation (12) can be performed by using the standard normal distribution function $\Phi(s)$ and its derivative, the probability density function $\phi(s)$ as,

$$P_f = \int_{-\infty}^{0} f(z)dz = \int_{-\infty}^{-\frac{\mu_z}{\sigma_z}} \phi(s)ds = \Phi(-\frac{\mu_z}{\sigma_z}) \tag{12}$$

Then the reliability index β is uniquely corresponds to the probability of failure, P_f, via Gaussian distribution, i.e.,

$$P_f = \Phi(-\beta) \tag{13}$$

The exact expression of the probability of failure for two variables R and Q is given in a form of convolution integral as,

$$P_f = \int_{-\infty}^{\infty} F_R(x) f_Q(x)dx \tag{14}$$

(5). Risk management

When the absolute safety is not possible, we, as engineers, have to make efforts to reduce the risk for the society. However we can reduce it to only a certain level since the uncertainty can not be eliminated, then we have to find out to transfer the risk to other systems. The insurance is one of the ways for the risk transfer. If the loss is evaluated in terms of the economical value, some kinds of financial management can be applied to the engineering risk problems.

The exact strength of the components can not be known and the maximum load intensity in future cannot be assessed without uncertainties. And yet if the probability of failure is less than 10^{-6} for example, we feel we are sufficiently secured in the ordinary life. Nevertheless, we face the possible failure which could cause a serious financial problems and need counter measures for the failure. Obviously we respond to the failure event according to the amount of risk.

Risk always deals with problems of the human loss or the casualty.　Such problems can not be compensated simply with the financial replacement, but have to be considered for the safety issue when the safety degree can be controlled to a certain extent.

Table 1 Probability of life loss

Cause for loss of life	Annual probability
Traffic accident	0.00008
Mountain climbing（international）	0.003
Airplane（crew）	0.001
Airplane（passengers）	0.0002
Fire	0.000001
Domestic accident	0.0001
Building（U.K.）	0.0000001
Building（Japan）	0.000001
Construction work	0.0004
Cancer due to nuclear accident（USA）	0.0001

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2. Annual Maximum Load Events

It is interesting to find analogical resemblances between structural safety and environmental safety, often called as environmental risk. Characteristics of loading or input play important roles in both safety problems. The social sciences have expanded their views to the environments based on some achievements for industrial pollutions such as Minamata disease or Kawasaki asthma. As we have many experiences of earthquakes, typhoons and heavy snows in Japan, views from residents, living persons and victims are needed to consider the structural safety.

For example when you compare the price of transportation, driving a private car is often cheaper than the train ticket. Cost of the road construction and traffic accidents should be counted as social cost for better environment. Some statistics tell that road construction and maintenance cost 2 million yen per car.

When a building is built, there is a risk for collapse due to natural environmental loads. Unfortunately this is not visible in many cases. And most buildings are demolished intentionally before its durable life limit because the probability of failure due to earthquakes or typhoons is very small, say 10^{-2} or 10^{-3} or even less. People can live without paying attention to such a possible collapse. But if you imagine the consequences of collapse, it is understood that any buildings have a negative property potential, which is not usually counted in a similar manner as the cost of water pollution or air pollution was not counted unless it becomes the social problems. If the failure probability is different for buildings, the society has to take actions according to the probability. This aspect will be discussed later in chapter 6.

(1). Characteristics of maxima

We have defined the probability distribution for a random variable. If we find special characteristics for the probability distribution, we should utilize such characteristics. Extreme value distributions are introduced by reflecting such characteristics. As we are interested in the maxima for loads, the right hand side tail of distribution has to be considered.

Some conditions are also discussed, e.g.,

(a). homogeneity, (b). independence, (c). sufficient number of data etc.

As the natural phenomena such as the wind, snow or earthquake, are influenced by many natural environmental parameters. Strictly speaking all the conditions mentioned above may not hold, but when we discuss maxima such as annual maxima, we can accept such conditions for the simplicity. The verification may be possible but only indirectly, as we want to make a use of probability models for prediction of future events.

(2). Return Period

The return period, R, is defined as the inverse of probability of exceedance as,

$$R = \frac{1}{P} \tag{0}$$

This concept is applicable to an independent random variable of an identical distribution. The probability of exceeding a certain value at the first time is considered. The probability of exceedance in the first year is P, then the probability of exceeding the value at the first time in the second year is the product of the probability of non-exceedance in the first year and the probability of exceedance, i.e., $(1-P)P$. By summing up to the infinity, it is clearly shown that the probability becomes 1, which means that if there is a probability of exceedance, an event greater than a certain value will occur in infinite time. This is confirmed by the mathematical expressions as,

$$\sum_{t=1}^{\infty}(1-P)^{t-1}P = \frac{P}{1-(1-P)} = 1$$

Then consider the expected year for the first time of exceedance, which is obtained in the following expression and defined as R,

$$\sum_{t=1}^{\infty} t(1-P)^{t-1}P = R$$

A mathematical manipulation is made.

$$R - R(1-P) = \sum_{t=1}^{\infty} t(1-P)^{t-1}P - \sum_{t=1}^{\infty} t(1-P)^{t}P = 1$$

Then the relation of equation (0) is confirmed.

(3). Derivation of Gumbel Distribution

When the parent distribution has a tail differentiable infinity times, the Gumbel distribution can be derived.

$X_{\max}$ can be described as the maximum of n independent variables, $X_1, X_2, \cdots X_n$.

$$\Pr[X_{\max} \le x] = \Pr[X_1 \le x \cap X_2 \le x \cap \cdots \cap X_n \le x]$$
$$= \Pr[X_1 \le x]\Pr[X_2 \le x]\cdots\Pr[X_n \le x] \qquad (1)$$

The cumulative distribution can be obtained for an independent and identical case,

$$F_n(x) = [P(x)]^n \qquad (2)$$

The probability density function can be obtained by differentiating eq.(2)

$$f(x) = n[P(x)]^{n-1} p(x) \qquad (3)$$

Further differentiation leads to,

$$f'(x) = n(n-1)[P(x)]^{n-2} p^2(x) + n[P(x)]^{n-1} p'(x)$$
$$= n[P(x)]^{n-2}\{(n-1)p^2(x) + P(x)p'(x)\} \qquad (4)$$

Consider the mode u_n. From the definition,

$$f'(u_n) = 0 \qquad (5)$$

By substituting $x = u_n$ into eq.(4)

$$\frac{(n-1)p(u_n)}{P(u_n)} = -\frac{p'(u_n)}{p(u_n)} \qquad (6)$$

When x is sufficiently large（the maximum of n variables is considered）,

$P(x) \to 1$, $p(x) \to 0$, $p'(x) \to 0$ then,

From L'Hospital's rule,

$$\text{For } x \to \infty, \quad \frac{p(x)}{1-P(x)} = -\frac{p'(x)}{p(x)} \qquad (7)$$

$x = u_n$ is also sufficiently large and by comparing eqs.(6) and (7),

$$P(u_n) = 1 - \frac{1}{n} \quad \text{is obtained.} \qquad (8)$$

Then Taylor's expansion for $P(x)$ is developed at $x = u_n$,

$$P(x) = P(u_n) + p(u_n)(x-u_n) + p'(u_n)\frac{(x-u_n)^2}{2!} + \cdots \qquad (9)$$

Each differential can be obtained one by one. From eqs(7) and (8),

$$p'(u_n) = -np^2(u_n) \qquad (10)$$

When it is differentiable infinity times, L'Hospital's rule can be applied one by one and by differentiating both the numeral and the denominator of eq(7),

For $x \to \infty$, $\dfrac{p'(x)}{p(x)} = \dfrac{p''(x)}{p'(x)}$ then by substituting eq(10),

$$p''(u_n) = n^2 p^3(u_n) \tag{11}$$

Similarly

For $x \to \infty$, $\dfrac{p''(x)}{p'(x)} = \dfrac{p'''(x)}{p''(x)}$ then,

$$p'''(u_n) = -n^3 p^4(u_n) \tag{12}$$

By substituting eqs(8), (10), (11) and (12) into eq(9) ,

$$P(x) = 1 - \frac{1}{n}\left[1 + \sum_{r=1} \frac{(x-u_n)^r}{r!}\{-np(u_n)\}^r\right] = 1 - \frac{1}{n} e^{-\alpha_n (x-u_n)} \tag{13}$$

where $\alpha_n = np(u_n)$

The distribution for the maximum of n variables can be obtained as the n-th power of eq(13).

Then the asymptotic extreme value distribution is developed, i,e, $n \to \infty$.

$$F(x) = \lim_{n\to\infty} F_n(x) = \lim_{n\to\infty}\{1 - \frac{1}{n} e^{-\alpha_n (x-u_n)}\}^n \tag{14}$$

Letting $\beta = e^{-\alpha_n (x-u_n)}$ ($\dfrac{\beta}{n}$ is sufficiently less than 1.)

$$F(x) = \lim_{n\to\infty}\{1 - \frac{\beta}{n}\}^n = e^{-\beta} = e^{-e^{-\alpha(x-u)}} \tag{15}$$

A distribution of double exponential form is derived and is called as Gumbel distribution [2-1].

Gumbel: $F(x) = \exp\{-\exp[-a(x-b)]\}$ for $-\infty < x < \infty$,

$\mu = b + 0.45\sigma$, $\sigma = 1.28 / a$

(4). Other forms of extreme value distribution

Frechet: $F(x) = \exp\left\{-\left(\dfrac{c}{x-\varepsilon}\right)^{\gamma}\right\}$ for $\varepsilon < x < \infty$,

$$\mu = c\Gamma\left(1 - \frac{1}{\gamma}\right), \quad \sigma = c\sqrt{\Gamma\left(1 - \frac{2}{\gamma}\right) - \Gamma^2\left(1 - \frac{1}{\gamma}\right)} \quad \text{for } \varepsilon = 0$$

Weibull: $F(x) = \exp\left\{-\left(\frac{w-x}{u}\right)\right\}^{\gamma}$ for $-\infty < x < w$

$$\mu = w - u\Gamma\left(1+\frac{1}{\gamma}\right), \quad \sigma = u\sqrt{\Gamma\left(1+\frac{2}{\gamma}\right) - \Gamma^2\left(1+\frac{1}{\gamma}\right)}$$

Kanda [2-2]: $F(x) = \exp\left\{-\left[\frac{w-x}{u(x-\varepsilon)}\right]^{\gamma}\right\}$ for $\varepsilon < x < w$

(5). Estimation of parameters

When we have data, we want to make a model by estimating parameters.
Moment method, Least squares method and Most likely-hood method are commonly used. Moment method for Gumbel distribution is very simple. For least squares method, you have to plot data on probability paper.

Hazen plot $F_i = (N - i + 0.5)/N$ may be good enough.

Thomas plot $F_i = (N - i + 1)/(N + 1)$ was recommended by Gumbel.

Gringorten plot $F_i = (N - i + 1 - a)/(N + 1 - 2a)$ is most reasonable.

Verification for the best plotting method can be made by Monte Carlo simulation for a known distribution model.

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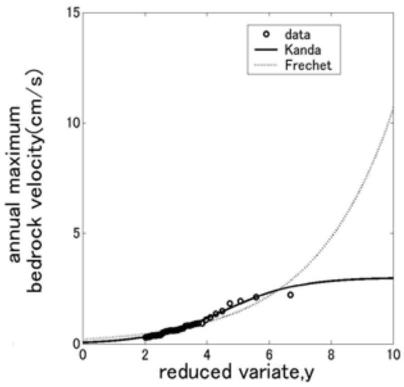
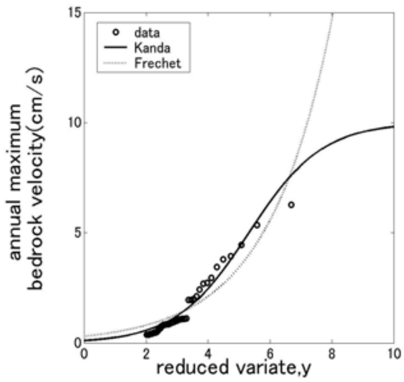
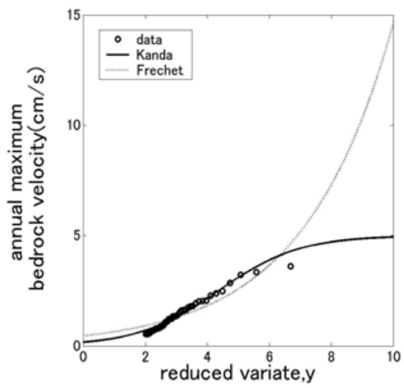


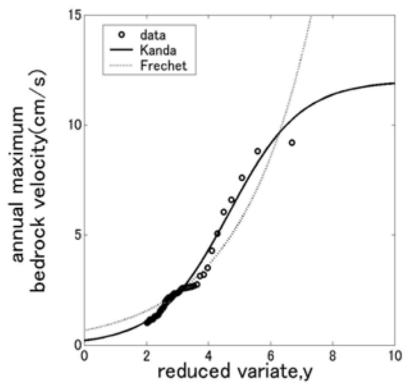
(a) Fukuoka



(b)Osaka



(c) Sendai



(d) Tokyo

Figure 2: Annual maximum bedrock velocities in four sites in Japan with Frechet distributions and Kanda distributions [2-3]

3. Probabilistic Earthquake Model

Earthquake ground motions are vibrations of the ground caused by the earthquake. Earthquakes are caused by a sudden rupture of fault where the stress was accumulated in the plate or the plate boundary. The energy was released at the ruptured area and was propagated to the surrounding soil layers.

- Plate tectonics: plate boundary earthquake (Great Kanto 1923), inland earthquake (Hyogoken-nambu 1995)
- Magnitude and Seismic Intensity: Surface wave magnitude, Moment magnitude, JMA intensity, MM intensity
- Active fault: Uemachi fault (Osaka, Occurrence Probability 0.00156/year), Kozu Matsuda fault (Kanagawa, 0.00118/year), Arakawa fault (Tokyo, 0.0003/year)
- Attenuation formula: Kanai, Joyner&Boore , Fukushima, Annaka etc.

$$A_{max} = f(M, \Delta) \quad (1)$$

where M : magnitude, Δ : distance

(1). Statistical approach for seismic hazard

Annual maximum PGA (Peak Ground Acceleration) or PGV (Peak Ground Velocity) values are estimated from historical earthquakes by an attenuation formula. The annual maxima are plotted on the probability paper to find a probability model. Frechet distribution has been used.

Return period conversion factor: $$\left(\frac{Q_2}{Q_1}\right) = \left(\frac{R_2}{R_1}\right)^{0.54} \quad (2)$$

where Q_i $(i = 1, 2)$ is the PGA corresponding to the return period of R_i. Basic values in AIJ load recommendation(1993) are available for $R_1 = 100$ (years).

(2). Earthquake occurrence model for seismic hazard

Gutenberg-Richter model: $\log n = a - b \cdot M$ (3)

Specific active faults with their occurrence probabilities have been available.

Access to http://www.j-shis.bosai.go.jp/ to find hazard map in J-SHIS

Characteristics of earthquake ground motion

- Intensity: PGA, PGV
- Duration time: Jenning's type envelope function
- Spectral characteristics: Power spectrum (stochastic measure), Response spectrum

(deterministic measure) (Acceleration, Velocity, Displacement)

(3). Earthquake resistant design:

Earthquake load can be modeled by a product of various parameters such as:

$$E_i = C_0 R_t D_S A_i W_i \qquad (4)$$

Base shear coefficient C_0 : base shear force divided by the weight above the story or the response acceleration divided by the gravity.

$C_0 = 0.2$ for the allowable stress design

$C_0 = 1.0$ for the capacity design (since 1981)

R_t factor: spectral characteristics are reflected according to the soil type

D_s factor: equivalent elastic limit for considering inelastic deformation (since 1981)

A_i factor: amplification factor for the i-th story.

(4). Seismic safety evaluation:

Access to http://ssweb.k.u-tokyo.ac.jp/ or http://ssweb-b.k.u-tokyo.ac.jp/

- a simplified prediction of seismic safety in Japan

According to the investigation for damages due to the Hyogoken-Nambu earthquake (1995) in the area of 15km by 5 km including the most severely damaged area, the collapse ratio of buildings built after 1981 is on the order of 0.5% for PGA of 6m/s^2 (Kanda, 1997[3-3]).

A typical value of PGA corresponding to 500 year return period (1/500 as the annual exceedance probability) in a high seismic activity zone in Japan is 4m/s^2 and the seismic load represented by PGA is normally assumed to follow the Frechét distribution. In order to roughly estimate the reliability of the building, the seismic load is converted to the equivalent log normal distribution by applying equation (1), which is a return period conversion formula proposed in Load Recommendation of Architectural Institute of Japan (1993) in order to obtain the seismic load in terms of 50 year maximum,

$R_i = \dfrac{1}{\tilde{F}_i}$, where $\tilde{F}_i$ is the exceedance probability in 50 years. Here we adopt

$Q_1 = 4m/s^2$, $R_1 = 10$ (corresponding to 50x10=500 years). In order to obtain the median value of 50 years maximum, $R_2 = 2$ is substituted into equation (2) because the median is the 50%-quantile meaning $\tilde{F}_2 = 0.5$. R_i in equation (2) is the return period in year in the original form, but because of the nature of power law expression, R_i can be the inverse of the exceedance probability in any reference period, such as 50 years in this case.

Then $Q_2 = 1.7$ is obtained as the median value of 50 year maximum of seismic load. An equivalent log-normal distribution at these two probability points, namely 50%-quantile and 90%-quintle may correspond to the distribution with the logarithmic mean of $\ln 1.7$ and the logarithmic standard deviation of 0.66.

As for the resistance of buildings, in a similar manner, by assuming the logarithmic standard deviation of 0.4 for the collapse ratio, the median is estimated as 16.8 m/s^2 from the collapse ratio of 0.5% for 6 m/s^2. Then the reliability index based on log-normal distributions for both the hazard PGA and the collapse ratio can be calculated as,

$$\beta = \frac{\ln 16.8 - \ln 1.7}{\sqrt{0.4^2 + 0.66^2}} = 2.96$$

This value corresponds to the probability of failure in 50 years of 0.0015. It is interesting to note that the probability of collapse estimated from the damage statistics is one order smaller than that regarded for the target safety criteria of design practice, i.e. the reliability index of 2 (Aoki et al, 2000[3-3]).

(5). Social system for seismic safety

- Seismic insurance

Table 1 Annual basic premium for RC or S structures. [3-4]

	Sapporo	Sendai	Tokyo	Nagoya	Osaka	Hiroshima	Takamatsu	Kitakyushu
Premium (%)	0.05	0.07	0.175	0.135	0.135	0.05	0.05	0.05

This table shows the current insurance premium for houses in Japan. The premium is basically applied to any houses in Japan and is considered to be rather higher than the expected annual loss estimates based on the seismic safety analysis.

- Indication of performance for fair trade

Reliability index defined by ISO2394 is one of example.

- Responsibility of Professionals
- Responsibility of individuals
- Promotion for seismic strengthening:

Subsidy for diagnosis, subsidy for strengthening

Non-retroactive for existing disqualified buildings

- Regulations and Standards

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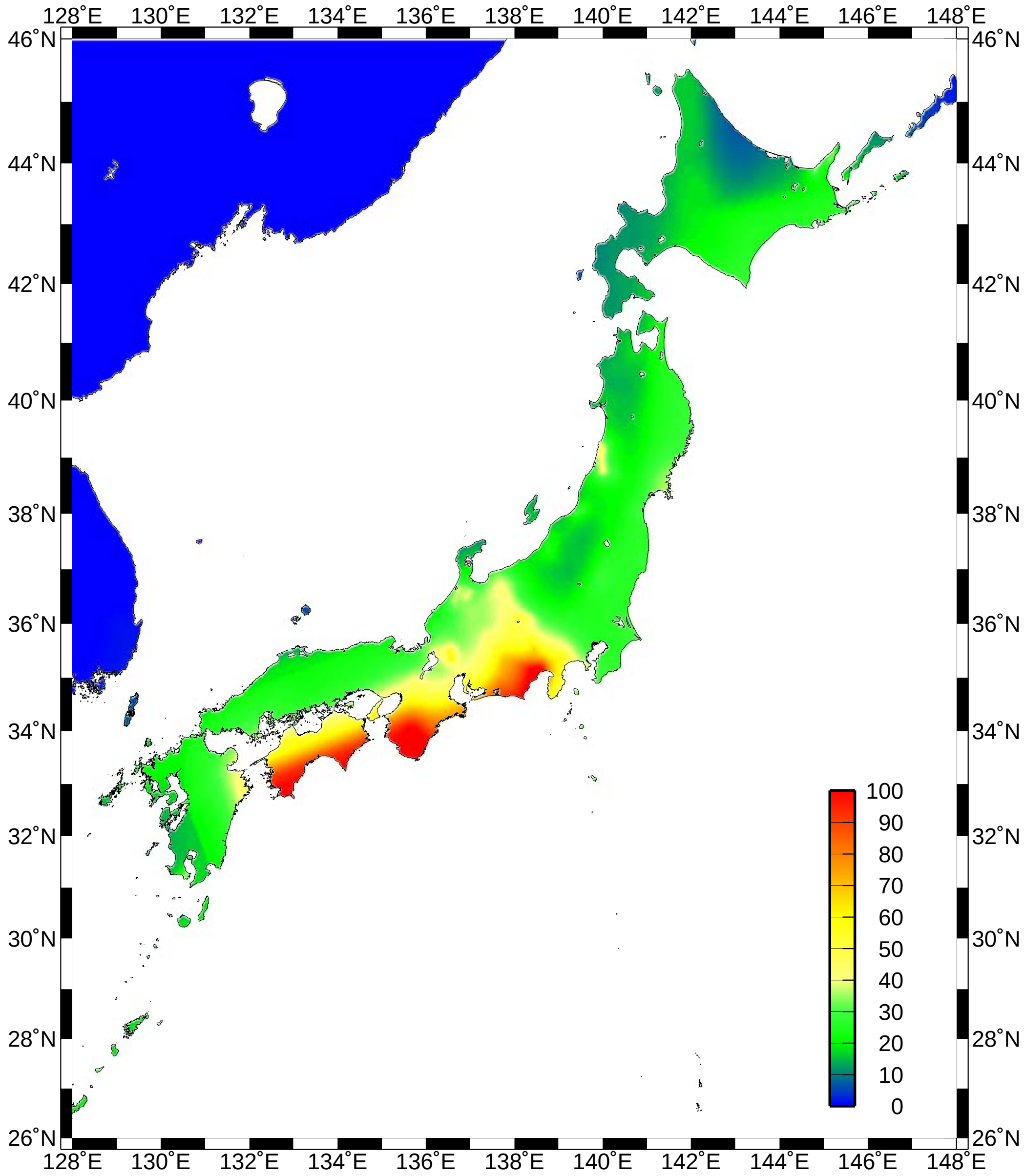

Figure 3. An example of seismic hazard map for10% in 50 year maximum PGV (in cm/s) cf. http://ssweb.k.u-tokyo.ac.jp/indexe.htm

4. FOSM Reliability

(1). Reliability Index based on AFOSM

The definition of reliability index by equation (11) on page 5 can be applied to non-linear and non-Gaussian cases. Another definition by equation (13) on page 5 is alternatively used and approximately correct for equation (11).

The limit state function can be defined for more than 2 random variables;

$G(x_1, x_2, \cdots) \geq 0$ desirable condition (safe) (1a)

$G(x_1, x_2, \cdots) = 0$ (limit state) (1b)

$G(x_1, x_2, \cdots) \leq 0$ non-desirable condition (failure) (1c)

All variables are normalized by subtracting the mean and dividing by the standard deviation as,

$$s_i = \frac{x_i - \mu_i}{\sigma_i} \tag{2}$$

Then $H(s_1, s_2, \cdots) = G(x_1, x_2, \cdots)$

A set of normalized variables, which satisfy the limit state condition equals 0, can be indicated as s_i^*, i.e.,

$$H(s_1^*, s_2^*, \cdots) = 0 \tag{3}$$

And the limit state function is linearized at this point by Taylor's expansion with the first derivative terms.

$$H(s_1, s_2, \cdots) = \sum_i \left(\frac{\partial H}{\partial s_i} \right)_{s_i = s_i^*} (s_i - s_i^*) \tag{4}$$

When variables are mutually independent, the mean and the standard deviation of H can be obtained as,

$$\mu_H = -\sum_i \left(\frac{\partial H}{\partial s_i} \right)_* s_i^* \tag{5}$$

$$\sigma_H = \sqrt{\sum_i \left(\frac{\partial H}{\partial s_i} \right)_*^2} \tag{6}$$

where the subscript * means partial derivatives at the set of s_i^*.

Then the reliability index according to the definition of equation (11) on page 5 can be written as,

$$\beta = \frac{\mu_H}{\sigma_H} = \frac{-\sum_i \left(\frac{\partial H}{\partial s_i}\right)_* s_i^*}{\sqrt{\sum_i \left(\frac{\partial H}{\partial s_i}\right)_*^2}} \tag{7}$$

The linearization is again applied to the square root of the sum of squares of the denominator term, by introducing the separation factor α_i.

$$\sqrt{\sum_i \left(\frac{\partial H}{\partial s_i}\right)_*^2} = \sum_i \alpha_i \left(\frac{\partial H}{\partial s_i}\right)_*$$

where $\alpha_i = \dfrac{\left(\frac{\partial H}{\partial s_i}\right)_*}{\sqrt{\sum_i \left(\frac{\partial H}{\partial s_i}\right)_*^2}}$ note that $\Sigma \alpha_i^{\,2} = 1$ (8)

Then the equation(7) becomes,

$$\sum_i \alpha_i \beta \left(\frac{\partial H}{\partial s_i}\right)_* = -\sum_i \left(\frac{\partial H}{\partial s_i}\right)_* s_i^* \tag{9}$$

It can be regarded that this equation is valid irrespective of $\left(\frac{\partial H}{\partial s_i}\right)_*$, then;

$$\alpha_i \beta = -s_i^* \quad \text{or} \quad s_i^* = -\alpha_i \beta \tag{10}$$

Now the reliability index is obtained.

$$\{\sum_i (s_i^*)^2\}^{\frac{1}{2}} = \{\sum_i (\alpha_i \beta)^2\}^{\frac{1}{2}} = \{\beta^2 \sum_i \alpha_i^{\,2}\}^{\frac{1}{2}} = \beta \tag{11}$$

The iteration is necessary as the set of s_i^* can not be appropriately chosen at first.

The procedure will be as follows,

① Assume that all s_i^* as 0. which corresponds to the mean of x_i.

② Partial derivatives $\dfrac{\partial H}{\partial s_i}$ at $s_i = s_i^*$ are calculated.

③ The separation factor α_i is calculated by equation (8).

④ By substituting equation (10) into equation (3) to solve β.

⑤ s_i^* is renewed by equation (10) then go to ②.

⑥ When the β value converges, the calculation is over.

The final s_i^* point is called the design point.

This iteration process makes the general solution for the non-linear limit state function. If the limit state function is linear, all partial derivatives are constant and the iteration is not necessary. When the variables are non-Gaussian, the equivalent Gaussian distribution at the design point with corresponding μ and σ, which are used for the normalization of x_i. Then the same procedure is applicable for the non-Gaussian case.

An alternative method to lead the same procedure is available to solve the distance of the limit state place to the origin in Figure 4 by applying the Lagrange multiplier method [4-1].

(2). Numerical Example for non-linear limit state function

$g = r - x^2$ for a limit state function and

$\mu_R = 250$, $\sigma_R = 25$ for resistance

$\mu_X = 10$, $\sigma_x = 2$ for load intensity

Normalized variables are $r = 25r' + 250$ and $x = 2x' + 10$, then

$$g = 25r' - 4x'^2 - 40x' + 150 \qquad \text{(e1)}$$

The partial derivatives are;

$$\frac{\partial g}{\partial r'} = 25 \text{ and } \frac{\partial g}{\partial x'} = -8x' - 40 \qquad \text{(e2)}$$

The first cycle,

① ② $\dfrac{\partial g}{\partial x'} = -40$ for $x'^* = 0$

③ $$\alpha_R = \frac{25}{\sqrt{25^2 + 40^2}} = 0.53 \text{ and } \alpha_X = \frac{-40}{\sqrt{25^2 + 40^2}} = -0.85 \qquad \text{(e3)}$$

④ Substituting $r'^* = -0.53\beta$ and $x'^* = 0.85\beta$ into equation (e1) =0,

$\beta = 2.72$ is obtained. (e5)

⑤ x'^* is renewed for the second cycle.

The second cycle,

① ② $\frac{\partial g}{\partial x'} = -8 \times 2.31 - 40 = -58.5$ for $x'^{*} = 0.85 \times 2.72 = 2.31$ (e6)

③ $\alpha_R = \frac{25}{\sqrt{25^2 + 58.5^2}} = 0.39$ and $\alpha_X = \frac{-58.5}{\sqrt{25^2 + 58.5^2}} = -0.92$ (e7)

④ Substituting $r'^{*} = -0.39\beta$ and $x'^{*} = 0.92\beta$ into equation (e1) =0, $\beta = 2.69$ is obtained.

⑤ x'^{*} is renewed for the third cycle.

⑥ The result shows the convergence. And $\beta = 2.69$.

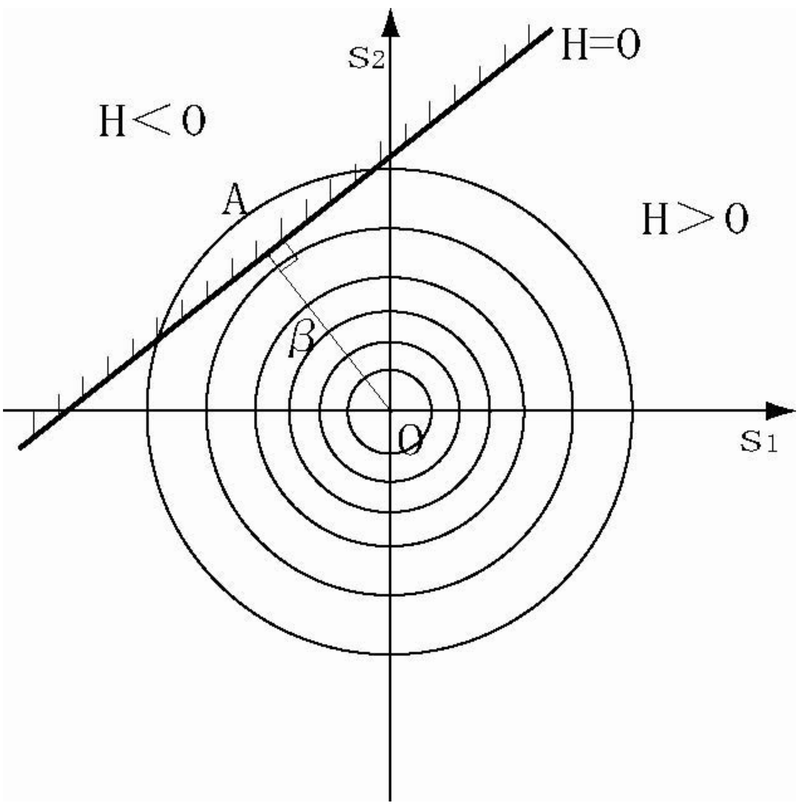

Figure 4. Limit state surface on normalized co-ordinates

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5. Limit State Design

(1). Fundamentals for limit state:

Both various kinds of performance demands and the grade for performances have to be clearly specified. In limit state design these kinds are defined by the limit states and the grade is defined by the reliability index.

The ultimate limit state is a limit state for the safety and the collapse for the system and the break for a member are typical ultimate limit states. Often the maximum load bearing capacity is used as an alternative conservative definition for limit state.

The serviceability limit state is a limit state for the ordinary use and the elastic limit may be conveniently used as no damages are expected within the limit. Often the deflection or acceleration are used for a specific demand such as the functionality of non-structural components or the human perception for motion.

Robustness is often referred for a required performance of structures, in particular for rare events. It is not treated as a quantitative measure in ISO2394 [5-1] and only recommended as an additional requirement. The definition of robustness can be a good theme of discussions in clarifying the nature of damages.

(2). Simple basic formulation for LSD [5-2]

Definition for the limit state function:

$$g = r - q \tag{1}$$

Then the probability of failure is described as:

$$P_f = \Pr[g < 0] = \Pr[r - q]$$

The reliability index corresponding to the P_f is given by the mean μ and standard deviation σ as,

$$\beta = \frac{\mu_g}{\sigma_g} = \frac{\mu_r - \mu_q}{\sqrt{\sigma_r^2 + \sigma_q^2}} \tag{2}$$

By introducing the separation factor α ,

$$\mu_r - \alpha_r \beta \sigma_r = \mu_q + \alpha_q \beta \sigma_q \tag{3}$$

The LRFD format can be written as,

$$\phi \mu_r \geq \gamma \mu_q \tag{4}$$

Or considering to the convenience of definition of design load and design strength currently used,

$$\phi' \hat{R} \geq \gamma' \hat{Q} \qquad (4\text{-}1)$$

where $\phi' = \phi \dfrac{\hat{R}}{\mu_R}$ and $\gamma' = \gamma \dfrac{\hat{Q}}{\mu_Q}$. $\hat{R}$ and $\hat{Q}$ are nominal values in current codes.

(3) Load factor for Log-normal variable

A common load factor format based on the log-normal load effect model is written as,

$$\gamma = \frac{1}{\sqrt{1+V_Q{}^2}} \exp(\alpha_Q \beta \sigma_{\ln Q}) \cdot \frac{\mu_Q}{Q_N} \qquad (5)$$

where Q is the seismic load effect, V_Q is the c.o.v. of Q, α is the separation factor, β is the reliability index $\sigma_{\ln Q}$ is the standard deviation of $\ln Q$, μ_Q is the mean of Q and Q_N is the basic load, often defined in terms of the return period. When the reduction factor considering the inelastic response is introduced simply as a multiplying factor such as Ds, the same form as eq.(4) in Chapter 3 can be used with modified the c.o.v. and the standard deviation considering the uncertainty of such a factor.

(4). Load factor considering inelastic responses

Now we propose a new form where the inelastic response characteristics are systematically considered. The seismic load effect in inelastic range can be expressed in terms of the equivalent elastic response, Q^*, as defined in the Figure 3. By carrying out sufficient number of inelastic response analyses, we can find the relationship between Q^* and the input ground motion intensity, a. Then such a relationship can be written as,

$$Q^* = \frac{m_R - 1}{m_a - 1} \cdot \frac{Q_y}{a_y}(a - a_y) + Q_y \qquad (6)$$

where $m_R = \mu_R / Q_y$ (resistance margin) and $m_a = a_u / a_y$ (input margin). μ_R is the mean of resistance in terms of Q^*, a_u and a_y are the PGA causing the response equal to the mean of yielding resistance Q_y and the mean of ultimate resistance μ_R.

For the log-normal variables of the resistance and the load effect, the reliability index β is obtained as,

$$\beta = \frac{\overline{\ln R} - \overline{\ln Q^*}}{\sqrt{\zeta_R^{\,2} + \zeta_Q^{\,2}}} \tag{7}$$

where $\bar{\ }$ indicates the mean and ζ indicates the logarithmic standard deviation. When eq.(6) is reduced to the linear relationship between $\ln Q^*$ and $\ln a$ as,

$$\ln Q^* \cong \frac{\ln m_R}{\ln m_a}(\ln a - \ln a_y) + \ln Q_y \tag{8}$$

This equation enables the transformation from a log-normal variable, a to another log-normal variable Q^*, then, [3-2]

$$\beta = \frac{\ln m_R - \frac{\zeta_R^{\,2}}{2} + \frac{\ln m_R}{\ln m_a}\left(\frac{\zeta_a^{\,2}}{2} + \ln\frac{a_y}{\mu_a}\right)}{\sqrt{\zeta_R^{\,2} + \left(\frac{\ln m_R}{\ln m_a}\zeta_a\right)^2}} \tag{9}$$

where μ_a is the mean of maximum PGA for a reference period, e.g. 50 years. Schematic diagram for eq.(6) and eq.(9), is shown in Figure 3. By assuming $a_y/\mu_a = Q_y/\mu_Q$, which is the relationship between the PGA and the elastic response, and neglecting the variation caused by the elastic response analysis, i.e. by assuming $\zeta_Q = \zeta_a$, eq.(9) can be rewritten as,

$$\alpha_R \beta \zeta_R + \alpha_Q \beta \frac{\ln m_R}{\ln m_a}\zeta_Q = \ln m_R - \frac{\zeta_R^{\,2}}{2} + \frac{\ln m_R}{\ln m_a}\left(\frac{\zeta_a^{\,2}}{2} + \ln\frac{Q_y}{\mu_Q}\right) \tag{10}$$

where α is the separation factor. Then the load factor design format may be written for the yield strength resistance rather than the ultimate resistance by equating μ_{Ry} to Q_y,

$$\exp\left(-\alpha_R \beta \zeta_R \frac{\ln m_a}{\ln m_R}\right)\mu_{Ry} = \frac{1}{\sqrt{1+V_Q^{\,2}}}\exp(\alpha_Q \beta \zeta_Q)\frac{1}{m_a}\mu_Q \qquad (11)$$

In comparison with the load factor format by eq.(5), $1/m_a$ corresponds to the inelastic performance factor Ds. The term $\dfrac{\ln m_a}{\ln m_R}$ in the resistance factor will be in the range of 0.5 to 2 according to the ratio of the fundamental frequency of the structure and the dominant frequency of the input motion depending on the type of hysteretic behavior.

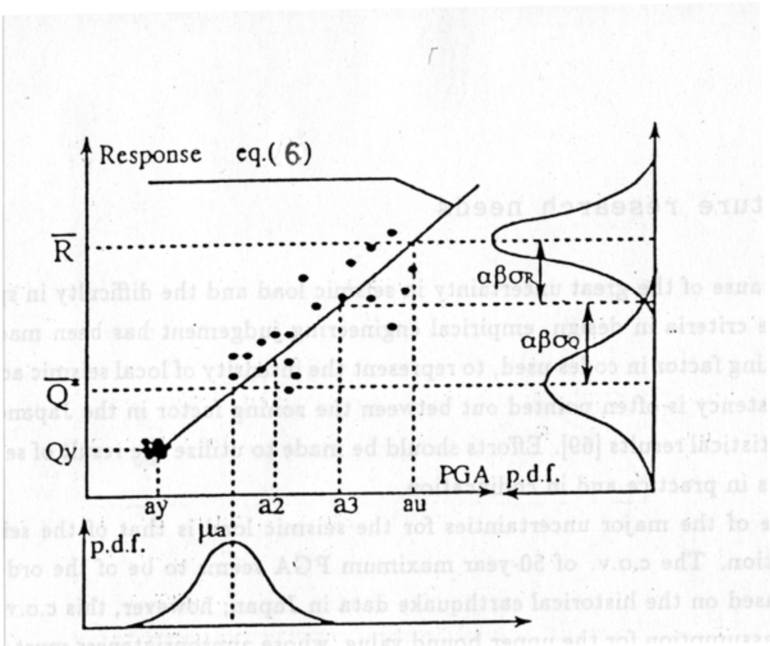

Figure 5. Schematic diagram for probabilistic inelastic responses

(5). Possible advantages for Limit State Design

AIJ has conducted a competition for Active Use of LSD in 1999 and 6 excellent proposals among 43 proposals are now available in a form of Book published in2004 [5-4]. Each proposal has a special theme for the use of LRFD. They are:

(1) Renovation of historical building – "conversation" with clients –

(2) Limit State Design with the deflection as the performance measure – a trial for quantification of seismic performance –

(3) Prevention of story collapse – Failure reliability of structural system –

(4) Longer life building – Design procedure considering the time axis –

(5) Revitatlization of traditional timber culture – LSD since there is a variability –

(6) Cost and performance – Design considering life cycle of building –

(6). Consideration on Load Factors

There are two procedures for load factors. When a <u>characteristic value</u> such as defined as a value corresponding to 100 year return period is used to define the <u>representative value</u> as used as the <u>basic value</u> in AIJ Load Recommendation (2004), load factors are estimated to satisfy the target reliability by considering the coefficient of variation of load effects and the separation factor. Load factor values are generally greater than 1.

The second procedure for load factors is that the unity load factor is used for a standard requirement case. Then a load factor may be used as an importance factor considering failure consequences of the structure. Different return periods will be used to define <u>representative values</u> for the serviceability limit and the ultimate limit. Some practical engineers prefer the second procedure to the original LRFD as they can consider the load intensity directly related to the exceedance probability, i.e. in a semi-deterministic manner. However it has to be noted that the return period only indicates the probability of exceedance of the load intensity but uncertainties of other parameters are not well formulated in the second procedure.

In the discussion of ISO draft for structural design framework, two procedures are adopted as alternative procedures [ISO/DIS 22111: Basis for design of structures – General Requirements]. In ISO 3010: Seismic Actions on Structures, two alternative sets of load factors are provided in annex. Tables are shown for the consideration.

Some more considerations are made for load factors by utilizing the return period conversion factor based on extreme value distributions.

Table A.1 – Example 1 for load factors $\gamma_{E,u}$ and $\gamma_{E,s}$, and representative values $k_{E,u}$ and $k_{E,s}$ (where $k_{E,u} \neq k_{E,s}$) due to ISO3010(1999) [5-5]

Limit state	Degree of importance	$\gamma_{E,u}$ or $\gamma_{E,s}$	$k_{E,u}$ or $k_{E,s}$	Return period for $k_{E,u}$ or $k_{E,s}$ $k_{E,s}$ の
Ultimate	a) High	1,5 – 2,0	0,4	500 years
	b) Normal	1,0		
	c) Low	0,4 – 0,8		
Serviceability	a) High	1,5 – 3,0	0,08	20 years
	b) Normal	1,0		
	c) Low	0,4 – 0,8		

Table A.2 – Example 2 for load factors $\gamma_{E,u}$ and $\gamma_{E,s}$, and representative value k_E

Limit state	Degree of importance	$\gamma_{E,u}$ or $\gamma_{E,s}$	k_E	return period for k_E
Ultimate	a) High	3,0 – 4,0	0,2	100 years
	b) Normal	2,0		
	c) Low	0,8 – 1,6		
Serviceability	a) High	0,6 – 1,2		
	b) Normal	0,4		
	c) Low	0,16 – 0,32		

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6. Optimum Reliability Concept

(1). Minimum Total Cost Principle [6-1]

The essential purpose of structural engineering is how to make a safe structure. While there is no 100% safety, only the structural engineering does not provide a sufficient answer to the question how safe is good enough. Ideally you can state that a good balance among the safety, the functionality, the economy and the aesthetics, but if this balance is due to the decision of individual engineer, such solution may not be technologically treated.

The balance between the safety and the economy can be obtained by the minimum total expected cost. This theme has been studied in Kanda laboratory since 1988. And the solution can be obtained relatively easily, once cost parameters are formulated and several proposals were reviewed in reference [6-2].

- Formulation

$$C_T = C_I + C_F P_f \tag{1}$$

where C_T is the total expected cost, C_I is the initial construction cost and C_F is the failure cost. The initial construction cost has been studied and a linear form is confirmed for various types of buildings [6-2].

$$C_I = C_0\left[1 + k\left(\frac{r_0}{\mu_Q} - 1\right)\right] \tag{2}$$

The failure cost estimation is one of important issue for risk management. In this simple formulation, it is treated as a subjective and deterministic quantity in terms of the reference value of the initial cost.

$$C_F = gC_0 \tag{3}$$

The optimum condition can be obtained from the minimization of eq. (1), i.e.,

$$\frac{dC_T}{dr_0} = 0 \tag{4}$$

Closed form solutions are available for some probability models. For example, Gaussian distribution case:

$$\beta_{opt} = \sqrt{2\ln\left(\frac{g}{\sqrt{2\pi}\,k\alpha V_Q}\right)} \tag{5}$$

Log-normal distribution case:

$$\beta_{opt} = -\alpha_Q \varsigma_Q + \sqrt{(\alpha_Q \varsigma_Q)^2 + 2\ln\left(\frac{g\sqrt{1+V_Q^2}}{\sqrt{2\pi}k\alpha_Q \varsigma_Q}\right)} \tag{6}$$

The role of engineers has been discussed with regards to the normalized failure cost, g, as it could be an alternative measure for the target safety [6-3]. For example a simplified relation for seismic design may be as simple as,

$$\beta = \log g + 1.7 \tag{7}$$

Of course it is not so simple for determining the target reliability for structures or the load factors for the structural design. Nevertheless it has to be stressed that the relation between the consequence evaluation and the required safety should be fundamentally reflected in the structural design.

Eq. (1) may be expanded by considering maintenance cost, failure cost due to several damage states and also the insurance coverage. Then,

$$C_T = C_I + C_M + \sum_i C_{Fi} \Delta P_{fi} + C_{ins} - \sum_j C_{Fins_j} \Delta P_{fj} \tag{8}$$

- optimum reliability depends on the hazard

One of interesting example can be shown only from consideration of eq. (2). When eq. (2) is rewritten for a site or region with a different mean maximum load effect μ'_Q, then the cost-up gradient and the reference initial cost also change as,

$$C_I = C'_0\left[1 + k'\left(\frac{r_0}{\mu'_Q} - 1\right)\right] \tag{9}$$

Eq. (9) should be exactly same as eq. (2) but with different parameters.

Therefore,

$$\frac{C_0 k}{\mu_Q} = \frac{C'_0 k'}{\mu'_Q} \quad \text{and} \quad C_0(1-k) = C'_0(1-k') \quad \text{then,}$$

$$\frac{C'_o}{\mu_Q}\frac{(1-k')k}{1-k} = \frac{C'_0 k'}{\mu'_Q} \quad \text{i.e.,} \quad \frac{\mu'_Q}{\mu_Q}\left(\frac{1}{k'} - 1\right) = \frac{1}{k} - 1$$

By simplifying with approximation,

$$\frac{\mu_Q'}{\mu_Q} \cong \frac{k'}{k}$$ is obtained, when $k << 1$

This relation explains that in a low seismicity region, the cost-up gradient becomes smaller and the higher reliability becoms optimum than that in a high seismicity region. When you can make your structure safer with less expenditure, you should have a safer structure. On the other hand if it is too expensive to provide a higher safety in a high seismicity region, you cannot afford to have a safety degree as that in the low seismicity region.

(2). Multiple levels of damages

The continuous damage model is considered here [6-4]. Then the normalized failure cost $g(x)$ can be written as,

$$g(x) = g_2 + \left(\frac{x - x_0}{1.0 - x_0} \right)^{\gamma} (g_1 - g_2) \tag{10}$$

where x is the load intensity measure and $x = 1.0$ corresponds to the collapse level. $x_0 = 0.2$, $g_1 = 2.3$ and $g_2 = 0.1$ are assumed here and $\gamma = 2$ is assumed. The ranges for $0.2 \le x \le 0.4$ and $0.4 \le x \le 0.7$ are regarded as the minor and moderate damages respectively. Computed results of the total expected cost is shown in Figure 6, by indicating the contributions of expected damage costs for damage levels.

The c.o.v. value is another key parameter for the optimum safety. Cases for 30%, 60% and 90% are shown in Figure 7. A higher load factor becomes the optimum for higher c.o.v. cases but the optimum reliability is lower. When the c.o.v. value is high, it is expensive to design a high target reliability index, and so we have to accept a relatively lower target reliability. Engineer should provide a good estimation for c.o.v. of loads at a specific site based on the most recent findings.

When the consequence of collapse is considered, the failure cost estimation is a key factor for the optimum degree of safety [6-3]. Figure 8 demonstrates the effects of the normalized failure cost, g , where the single damage state is considered. The minimum requirement for the collapse could be $g = 1$, but such a lowest standard may not be accepted by majority of residents. A higher g value may be claimed by the neighbor people but reasonable failure consequences should be discussed to have a consensus.

Another factor to be considered is the design service lifetime of the structure. The developer or investor considers a relatively short time since they will expect profit from the investment in a short range of time such as 5 or 10 years. On the contrary, people who want to live for their whole lives or leave properties for their descendants consider a relatively long time such as 50 years or 200 years. The effect of such time span for the probability is certainly of some significance. The results are shown in Figure 9. The discount rate may be considered with some uncertainty for the future time discussion.

(3). Evaluation of Environmental Impacts

The total life-cycle cost concept can be simply expanded to the CO2 emission evaluation. Some of results show that the lower safety becomes optimum as the structural portion has a higher contribution in the CO2 emission than the cost. It is discussed how to combine these different solutions [6-4].

There are more aspects for the evaluation of environmental impacts rather than the CO_2 emission evaluation. It is an open question how to minimize the environmental impact in construction.

Since the concern of global environment increases, the CO_2 emission may be an alternative measure to the monetary cost. The structural portion of initial cost is in the order of 25%, while the structural portion of CO_2 emission is in the order of 50% which is the ratio of the weight.[Kanda and Kanda, 2002] Then the gradient of initial cost to the load factor may be doubled then the optimal design safety will be lowered for the case of minimization of total CO_2 emission. This is demonstrated in Figure 10. The vertical axis indicates the cost and the CO_2 emission normalized by the reference value at load factor 1.

As demonstrated in figures with various conditions for parameters, the way of individual considerations causes the difference in the optimal safety as each individual has a different estimation of parameters. Stakeholders will have different advantages and disadvantages due to the degree of safety. The local authority could play a role of judge rather than checking the conformity to the regulation. Once the consensus is made, this will be an ideal way of acceptance of the safety requirements in a community.

Figures shown here are based on a simple model and only examples. Nevertheless influences of parameters can be seen in a realistic manner. If more specific information is available, models can be replaced or improved. Once the framework of the safety role in a form such as eq.(1), the meaning of safety will be understood by such analytical examinations to find influences of specific quantitative demands such as the damage levels, failure consequences, service lifetime and so on.

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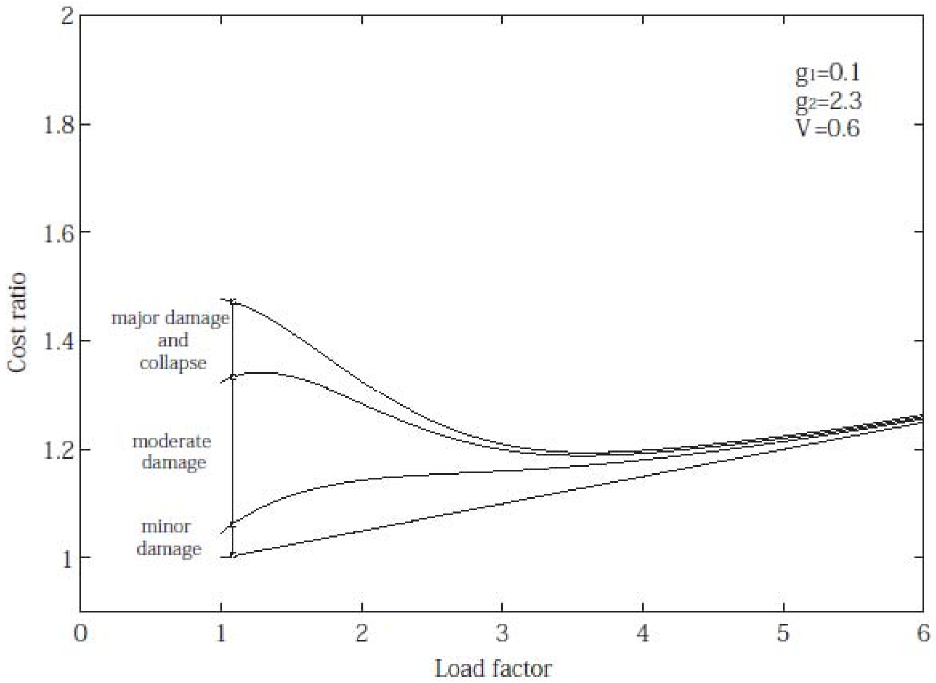

Figure 6: Optimal design safety with expected damage costs[6-6]

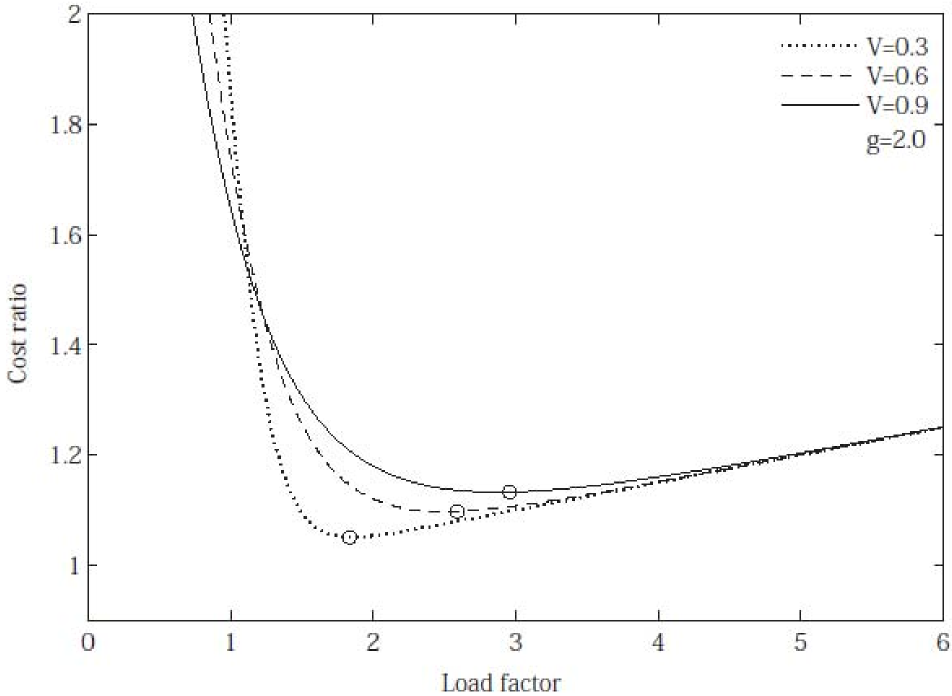

Figure 7: Optimal design safety with various c.o.v. cases [6-6]

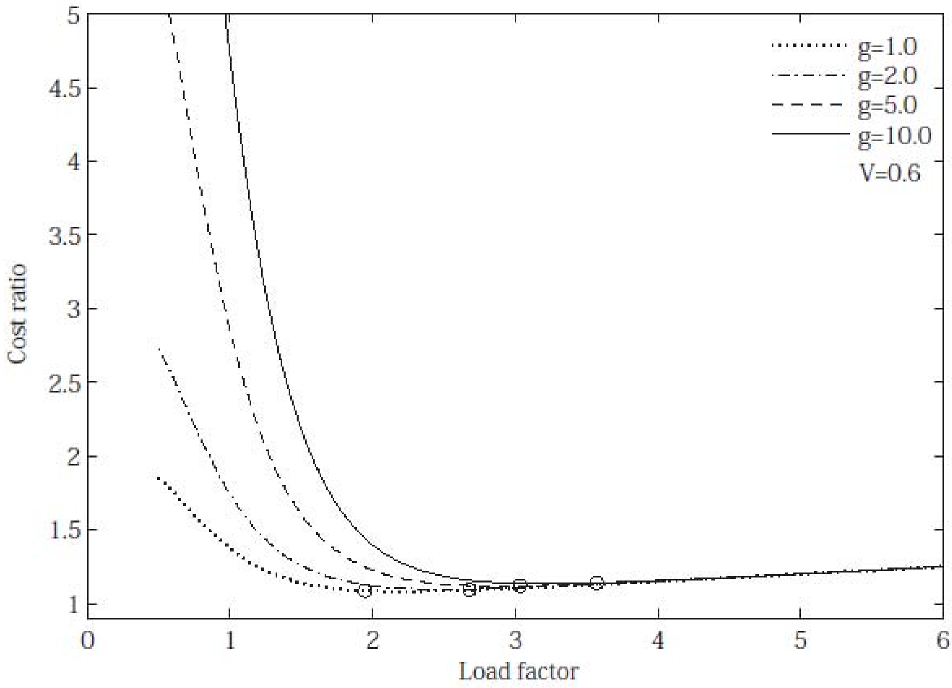

Figure 8: Optimal design safety with various normalized failure cost [6-6]

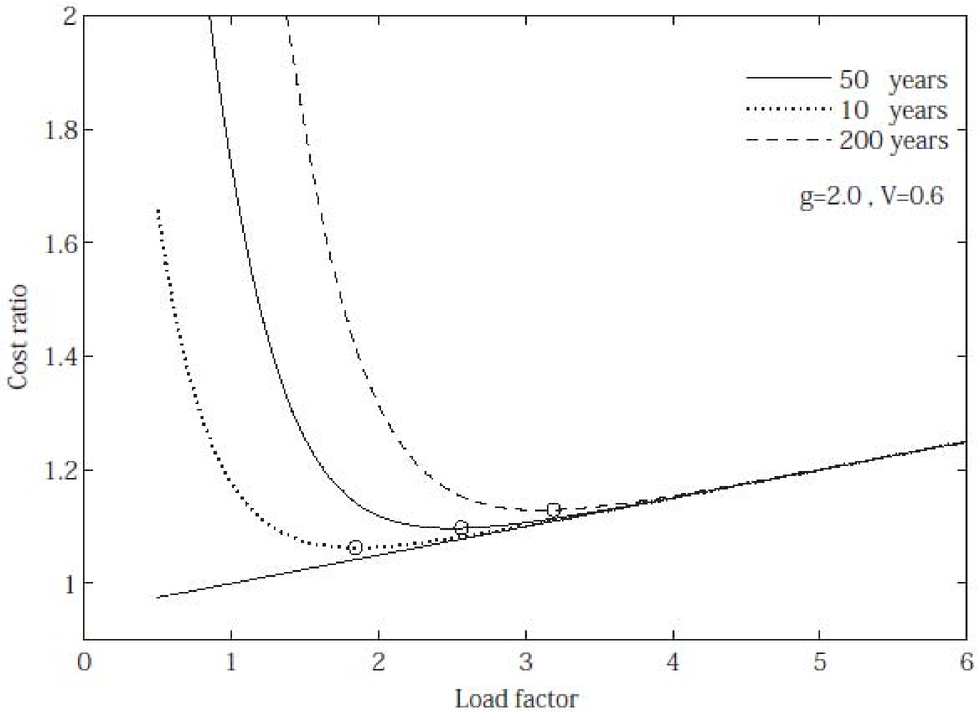

Figure 9: Optimal design safety with various service lifetimes [6-6]

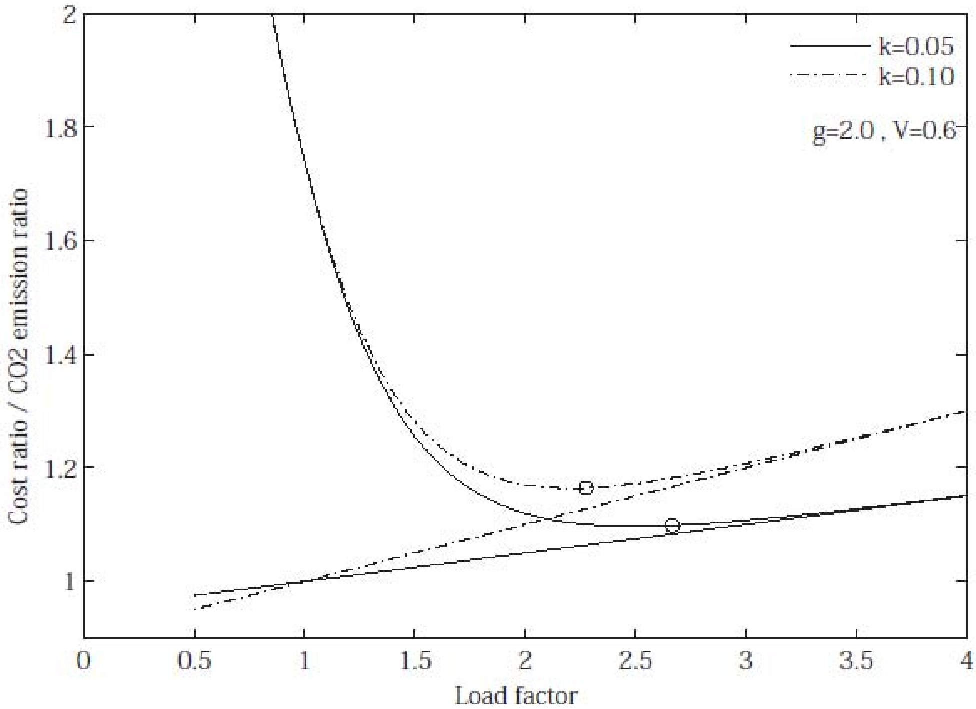

Figure 10: Optimal design safety for total cost and total CO_2 emission [6-6]